Financial Markets and the Real Economy: a statistical field perspective on capital allocation and accumulation

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Purpose of the paper

- We have previously developed and applied a field formalism, derived from statistical physics, to economic models with large number of agents
- Compared to standard approaches (representative agent, ABM, mean field):
  - No simulations needed
  - collective states are derived directly from the form of interactions between agents rewritten in terms of fields
  - Possibility to study impact of collective states on individual agents

Purpose of the paper

- Provide a step-by-step method to directly translate a classical economic framework with a large number of agents into a field-formalism model

- Apply this methodology to model the interactions between a large number of:
  - Investors, seen as the financial market
  - Producers, seen as the real economy

- Study capital allocation and accumulation resulting from these interactions in:
  - A static environment
  - A dynamic environment

Representative agent

One optimizing agent aggregates for many behaviours. Neglects or averages interactions.

Large number of agents

For a large number of agents:

The motion of one agent depends on all agents dynamics.

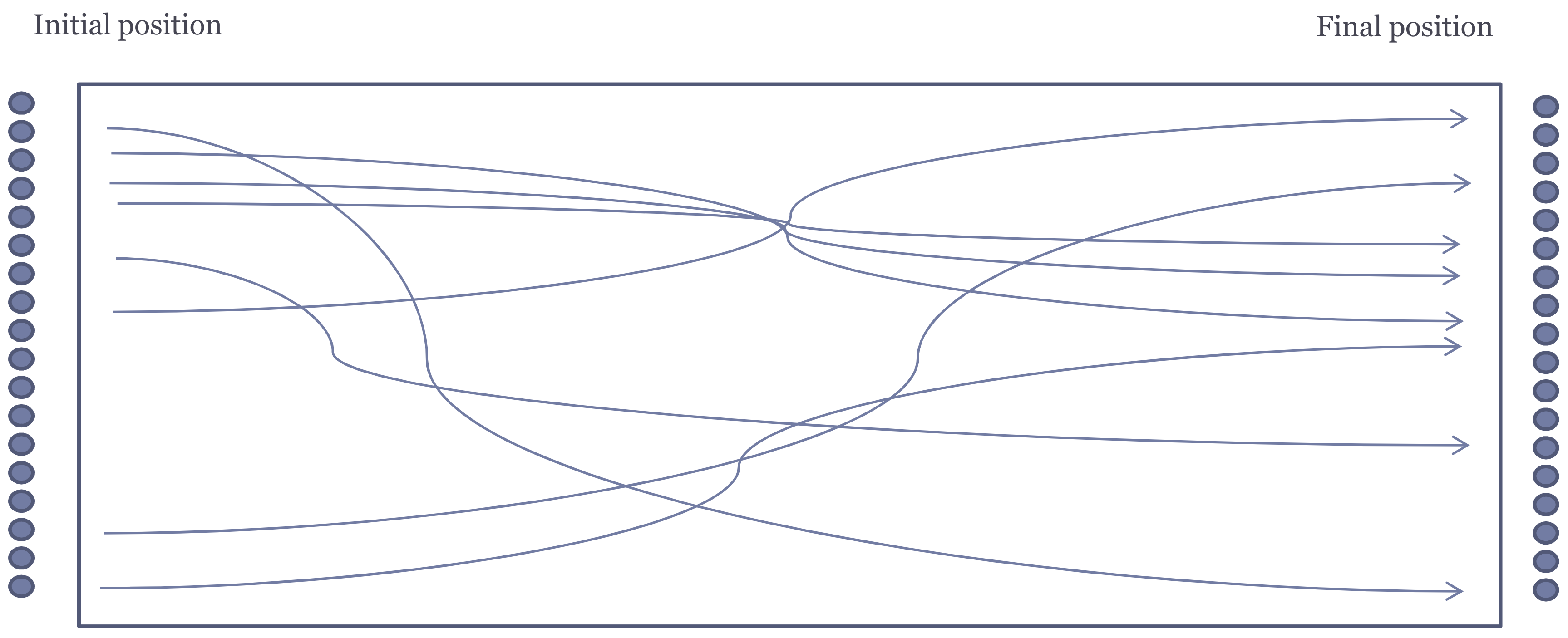
In a stochastic context, infinite number of trajectories for the N agents.

Each N-agent trajectory is a possible state of system, a « collective state. »

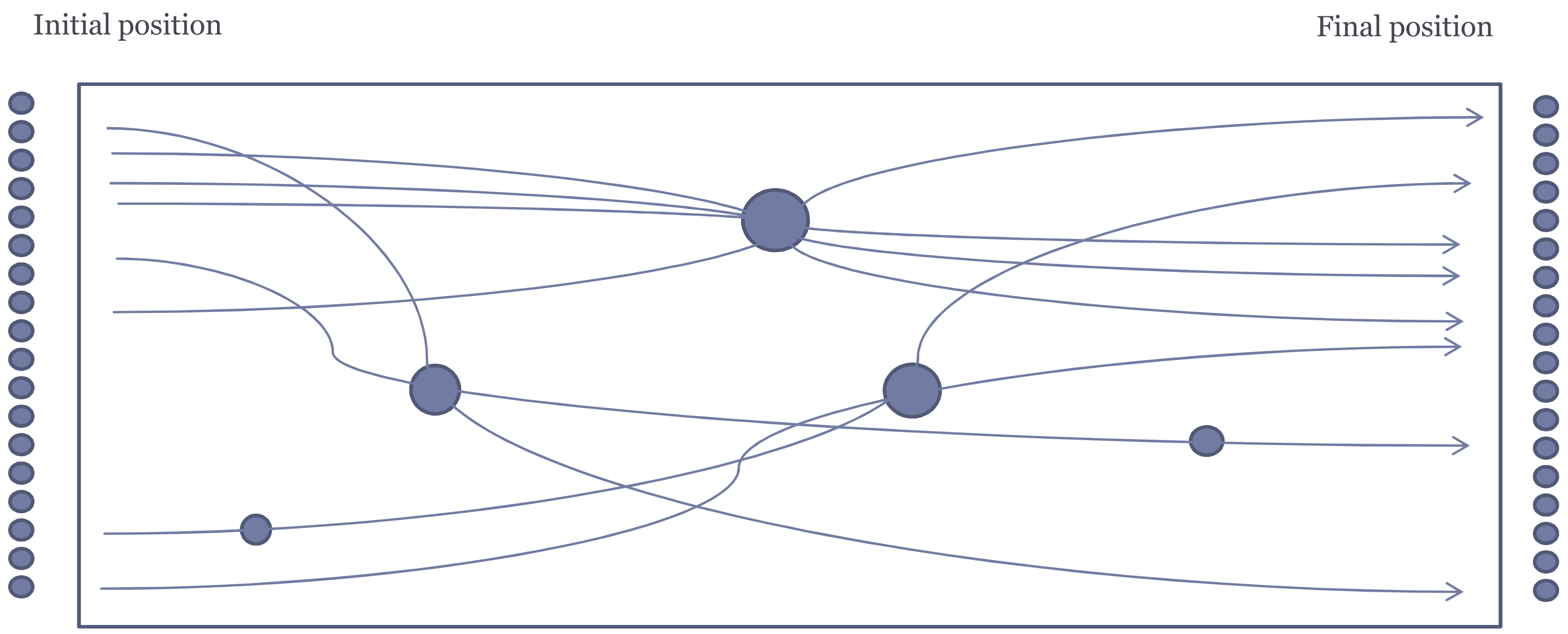
The trajectories arise in the abstract space of all economic variables (capital, consumption…)

A probability can be associated to such a state

Collective state in terms of trajectory



Collective state in terms of trajectory



Collective state in terms of density

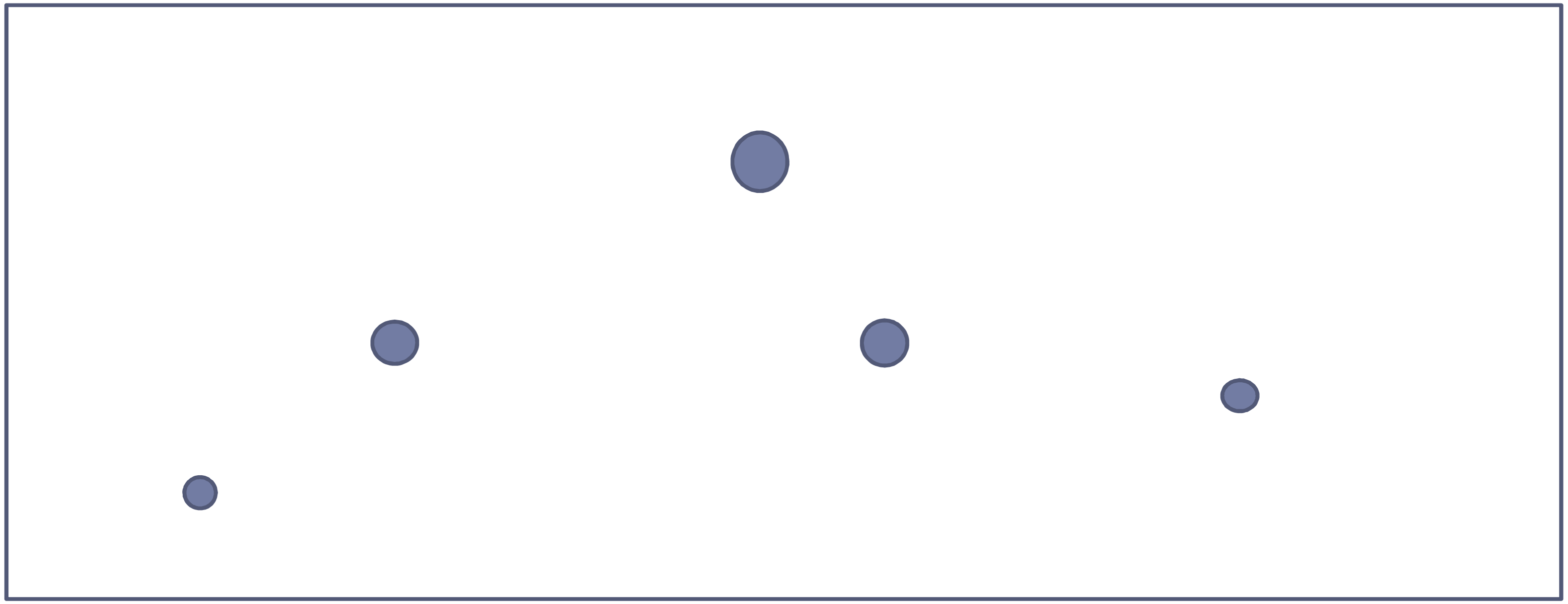
• Repeating the same trajectories allows to consider densities of agents with respect to the economic variables.

• Each point in the state space is crossed by a certain number of agents. This yields the density of agents for any given value of the economic variables.

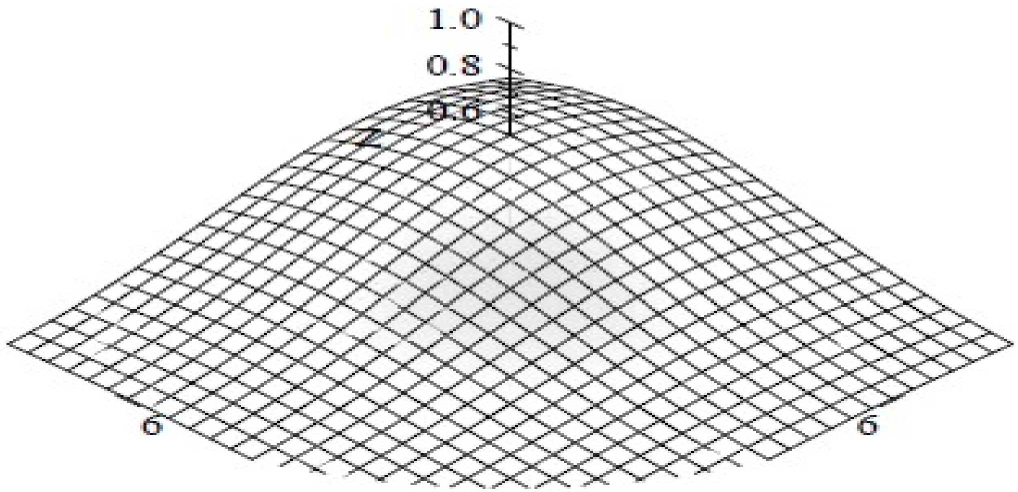
Collective state in terms of density

Initial position

Final position



Collective state in terms of density



Infinite number of possible collective states

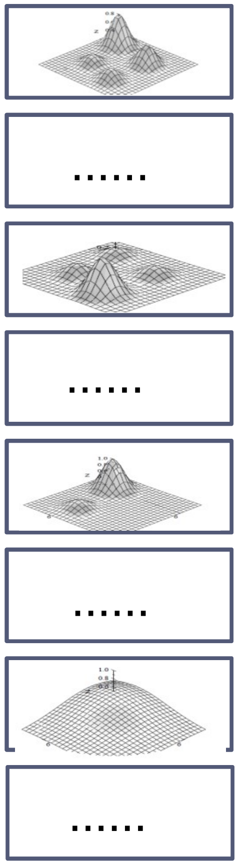
The whole possible trajectories induce many possibilities for collective states

A probability can be associated to each collective state (a sum over trajectories probability)

Summed up by the following pictures.

Field: the set of collective states

Realizations

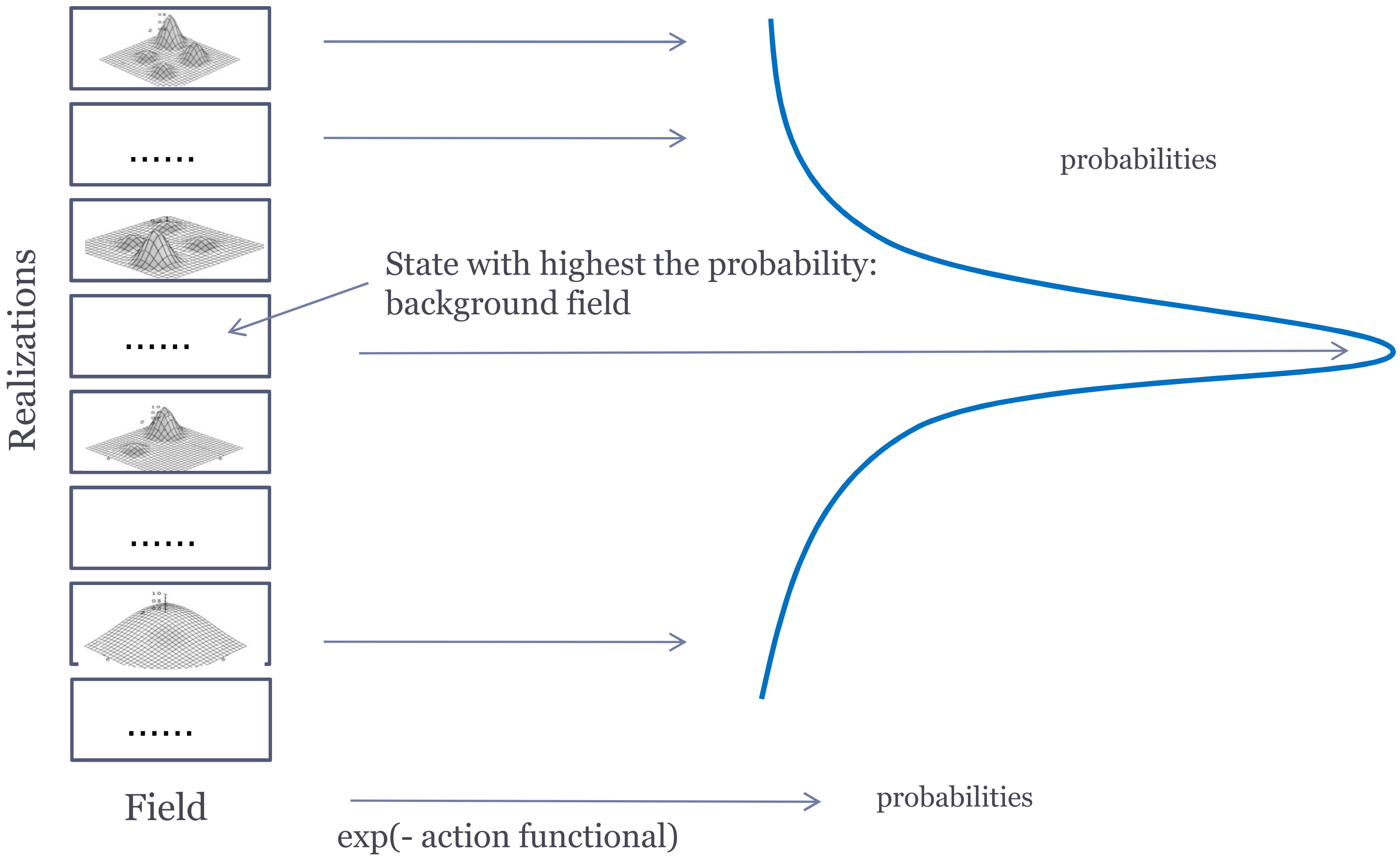


Field

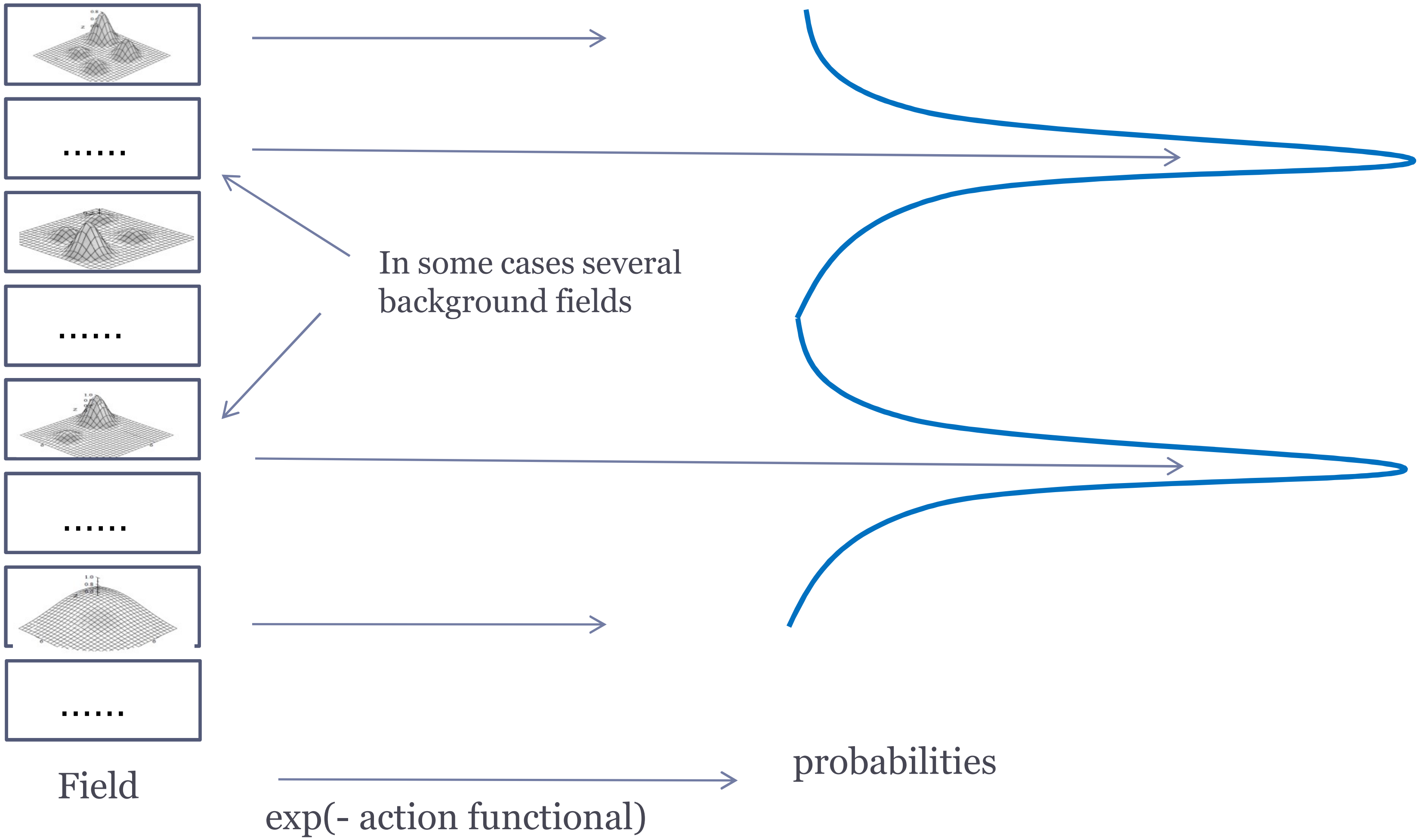
Collective states: the infinite realizations of a field

A random variable whose realization describes the possible collective states

Field: the set of collective states



Field: the set of collective states



Field + action functional = probability

The action functional associates to each collective state, a realization of the field, a probability

This translate the initial microeconomic model in terms of field.

The most likely realization(s) is (are) called the expression of the field, or the background field. It encompasses the m

Collective states vs. representative agent

Nothing ensures that the background field corresponds to copies of representative agent.

Finding the most likely collective state : the field formalism

- To study collective states, we translate the system of trajectories in terms of field.
- Allows to derive the most likely collective state: Background field
- Once the collective state is found, yields the averages of economic variables among agents.
- Also allows to come back to the individual dynamics conditioned by the background field.

Field formalism for a large number of agents

- Method developed previously by the authors
- Translate a dynamic system with a large number of agents into a statistical field model
- The N-agents dynamics is described by:
  - a “field”: an abstract function that encodes the agents
  - a function of this field: the “action” of the system that encompasses the dynamic system

Field formalism for a large number of agents

A system of dynamic equations:

$$\frac{d\mathbf{A}_i(t)}{dt} - \sum_{j,k,l\ldots} f\left(\mathbf{A}_i(t), \mathbf{A}_j(t), \mathbf{A}_k(t), \hat{\mathbf{A}}_l(t), \hat{\mathbf{A}}_m(t)\ldots\right)$$

Plus eventually some optimization problems (utility…)

Translation

Are replaced by a field ψ(A), and a field action S(ψ):

- The field ψ(A) keeps tracks of the set of agents:
  The dynamic variable A becomes the argument of the function
  One field for each type of agent
- The field action S(ψ) encodes agents' dynamics and interactions
  Its form depends directly on the dynamic equations

Field formalism for a large number of agents

- The solutions to the minimization equations of the action functional S(ψ) are called the ***background fields*** of the system

- They describe the potential equilibria of the system. They:
  - Characterize the collective states of the system
  - May be used to compute average quantities of the system
  - Structure the interactions between agents
  - Condition individual dynamics

- Once the background field found, expanding the action functional S(ψ) around this background yields the individual dynamics of generic agents in a given background field

Field formalism Advantages

The field formalism allows to:

- Deal analytically with the full system

- Study the emergence of some particular states of the system, the “background fields”, in which individual agents evolve

- Describe the collective behaviors of the system for a large number of agents

- Keep track of individual dynamics and describe generic emerging agents

This field formalism also allows a mutual interpretation between micro and macro levels.

Application: microeconomic framework

- Two groups of agents: producers and investors
- Producers represent the real economy
- Investors represent the financial markets

Application: microeconomic framework

- Firms
  - Large number of firms in different sectors X
  - Compete by producing differentiated goods
  - Are endowed with physical capital K
  - Physical capital depends on the capital lent by investors
  - May shift between sectors to improve their returns and attract investors
  - Reward their investors:
    - Pay dividends
    - Through the valuation of their stock prices

Application: microeconomic framework

- Investors:
  - hold financial capital $\hat{K}$
  - allocate it between firms across sectors according to:
    - Investment preferences
    - Expected returns of firms R(K,X)
    - Stock prices variations on financial markets
  - Move along sectors based on firms' expected long-run returns
  - Increase their capital through dividends and stock prices

Application: microeconomic framework

The dynamics follows the following pattern:

1. Investors allocate their capital between firms
2. Each firm uses its capital to produce one good
3. Short-term returns are generated
4. Capital is returned to investors

1. Financial capital allocation

- Investors allocate their capital to producers:

$$\hat{K}_j^{(i)}(t) = \left(\frac{F_2(R(K_i, X_i))\, G\left(X_i - \hat{X}_j\right)}{\sum_l F_2(R(K_l, X_l))\, G\left(X_l - \hat{X}_j\right)} \hat{K}_j \right)(t)$$

Share of capital invested in firm *i*. Depends on expected returns

Capital invested in firm *i* by firm *j*

2. Firms' disposable capital

- Producers' capital is the sum of capital invested

$$K_i(t+\varepsilon) = \sum_j \hat{K}_j^{(i)} = \sum_j \frac{F_2(R(K_i(t), X_i(t)))\, G\left(X_i(t) - \hat{X}_j\right)}{\sum_l F_2(R(K_l(t), X_l(t)))\, G\left(X_l(t) - \hat{X}_j\right)} \hat{K}_j(t)$$

Capital invested in firm *i* by firm *j*

Capital at the beginning of the period

3. Short-term returns

- The short-term returns of a firm are composed of:
  - Dividend r_i that is both:
    - firm-dependent
    - Sector-X and capital-K dependent

$$r_i = r\left(K_i, X_i\right) - \gamma \sum_j \delta\left(X_i - X_j\right) \frac{K_j}{K_i}$$

  - Variations in stock prices

$$\frac{\dot{P}_i}{P_i} = F_1\left(\frac{R\left(K_i, X_i\right)}{\sum_l R\left(K_l, X_l\right)}\right)$$

- R is the expected long-run return of the firm
  It depends on K and X

4. Financial payoffs

$$\hat{K}_j(t+\varepsilon) - \hat{K}_j(t) = \sum_i \left(r_i + \frac{\dot{P}_i}{P_i}\right) \hat{K}_j^{(i)} = \sum_i \left(r_i + F_1\left(\frac{R(K_i, X_i)}{\sum_l R(K_l, X_l)}, \frac{\dot{K}_i(t)}{K_i(t)}\right)\right) \hat{K}_j^{(i)}$$

Return of firm *i*

Capital invested by investor *j* in firm *i*

Financial capital variation between two periods

Dynamics within sectors' space

- The model is closed by considering that both producers and investors move within the sectors' space towards higher returns (equations given in the text)
- Main characteristics:
  - Producers are:
    - Driven by the perspective of higher returns
    - Deterred by competition in the targeted sector
  - Investors are driven by:
    - Perspective of higher returns relative to the neighboring sectors
    - Stock prices variations

Framework : synthesis

- Two variables shape the landscape and condition the form of the state of the system:
  - Short-term returns:
    - Dividends r
    - Price variations

  Depend on sectors, capital invested, competition…

  - Expected long-term returns R:
    - Impact stock prices: R depends on sectors X and capital invested K

- Variations of these quantities permanently modify the collective state of the system

  They induce a dynamics in potential equilibria

Field translation of the system

We translate the system in terms of fields

- The field translation involves:
  - Two fields:
    - One for the real economy: $\Psi(K, X)$
    - One for the financial markets: $\hat{\Psi}\left(\hat{K}, \hat{X}\right)$
  - A field action functional S, from which we derive the collective state of the system

Field translation of the system

$$\begin{aligned}
S = & -\int \Psi^{\dagger}(K,X)\left(\nabla_X\left(\frac{\sigma_X^2}{2}\nabla_X - \nabla_X R(K,X)\,H(K)\right) - \tau\left(\int \left|\Psi(K',X)\right|^2 dK'\right)\right. \\
& \left. + \nabla_K\left(\frac{\sigma_K^2}{2}\nabla_K + u\left(K,X,\Psi,\hat{\Psi}\right)\right)\right)\Psi(K,X)\,dK\,dX \\
& -\int \hat{\Psi}^{\dagger}\left(\hat{K},\hat{X}\right)\left(\nabla_{\hat{K}}\left(\frac{\sigma_{\hat{K}}^2}{2}\nabla_{\hat{K}} - \hat{K} f\left(\hat{X},\Psi,\hat{\Psi}\right)\right) + \nabla_{\hat{X}}\left(\frac{\sigma_{\hat{X}}^2}{2}\nabla_{\hat{X}} - g\left(K,X,\Psi,\hat{\Psi}\right)\right)\right)\hat{\Psi}\left(\hat{K},\hat{X}\right)
\end{aligned}$$

Field describing producers

Field describing investors

Functions *u*,*f*,*g*,*H* encode the micro-framework (given in the text)

Resolution of the field model

- The paper computes the background fields, or the “collective states”, for the real economy and the financial markets

- From a sector perspective, the collective states determine:
  - Capital average distribution per sector
  - Firms' concentration within sectors

  These quantities depend on external parameters, such as:
  - Changes in expected returns
  - Changes in dividends (technological advances)
  - Their dynamics, when these external conditions evolve

Results : static environment of the field model

- put differently, the collective configurations are characterized, at each point of the sectors' space, by:
  - The equilibrium average capital
  - The number of firms

Results : static environment

- Sectoral capital accumulation depends on the environment, i.e.:
  - Short-term returns (dividends and price variations)
  - Expected long-term returns
  - Relative expected long-term returns: sectoral accumulation is not local, it depends on the landscape

- There is a partial trade-off between these variables

- The number of firms per sector depends on:
  - the average level of capital invested in the sector
  - The expected long-term return

Results : static environment

For each sector, three possible patterns of capital accumulation emerge:

- Pattern I:
  - Dividends in short-term returns are determinant for accumulation
  - Sectors with few firms and low average capital
- Pattern II:
  - Sectors' short and long-term returns drive capital accumulation
  - Sectors with intermediate-to-high capital firms
- Pattern III:
  - Higher expectations of long-term returns drive massive inputs of capital
  - Instability in capital accumulation may arise among and within sectors: thresholds effects appear in average capital

Results : static environment

- The equilibrium may be unstable:
  - Changes in parameters or expectations may induce changes in portfolio allocation.
  - May leave some sectors deserted
- At a macro-timescale:
  - Any deviation from an equilibrium drives a sector towards the next stable equilibrium, zero included
  - When there is none, towards infinity
- This instability is relative and context-dependent:
  - Variations of parameters in some sectors may propagate to other sectors

Results : dynamic environment

To account for this systemic instability, we adopt a wider approach to our model.

We consider a dynamic system involving:

- Average capital per sector
- Endogenized long-term expected returns (most volatile parameter)

Results : dynamic environment

- Average capital per sector interact with:
  - One another
  - Long-term expected returns

- This dynamic system differs from those in standard economic:
  - In economics the dynamics is usually studied around a static equilibria
  - We consider the dynamic interactions between potential equilibria and expected long-term returns

Results : dynamic environment

- Some solutions of this dynamic system are oscillatory:

  Changes in one or several sectors may propagate over the whole sectors' space

- Pattern III sectors (high capital, high expected return):
  - Favored by fluctuations when expectations are highly sensitive to capital variations
  - These sectors drive capital from neighboring sectors

Conclusion

- This example shows that field formalism allows detailed analysis of systems with large number of agents
- Next paper: step-by-step method to compute agents dynamics within background states

Thank you for your attention!